

Understanding and Predicting Changes in Population Size – Exponential and Logistic Mathematical Models vs. Complex Reality¹

In biology, a **population** is a group of individuals of the same species that live in the same area at the same time. In this activity you will analyze a variety of examples to learn about how and why population size changes.

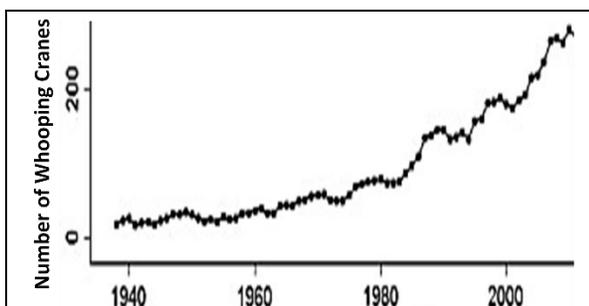
Recovery of Endangered Species – Why does it take so long?

An **endangered species** is a type of animal or plant that is at risk of becoming extinct because of low numbers and environmental threats. For example, due to hunting and habitat loss, the number of wild whooping cranes decreased from an estimated 10,000 before European settlement to about 20 in the 1940s.

Beginning in the 1950s, conservation efforts included protection against hunting and creation of wildlife refuges for whooping cranes. As a result, the one surviving population of wild whooping cranes increased from about 20 in the 1940s to more than 500 in 2021.



How tall do you think a whooping crane is? To learn more and see whooping cranes in action, go to [this link](#) or [this link](#).



Dates	Average number of whooping cranes	Increase during the previous 20 years
1950-1955	24.4	
1970-1975	53.0	28.6
1990-1995	138.0	72.0
2010-2015	290.6	152.6

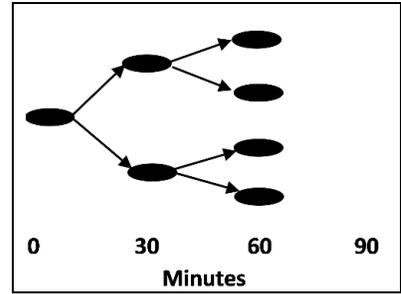
- 1a. For this population of whooping cranes, did population size increase by the same amount each year from 1950 to 2015? yes ___ no ___
- 1b. On average, when was the increase in population size bigger?
In earlier years ___ In more recent years ___
- 1c. Circle the data that support your answers to questions 1a and 1b.
2. What are two possible reasons why the increases in population size have been bigger in more recent decades?

We will return to an analysis of the recovery of endangered species, but first you will learn how populations grow by analyzing simpler examples of population growth.

¹ By Dr. Ingrid Waldron, Department of Biology, University of Pennsylvania, © 2023. This Student Handout, a shorter version without equations, and Teacher Notes (with instructional suggestions and background biology) are available at <https://serendipstudio.org/exchange/bioactivities/pop>. The recommended links under the picture of the whooping crane are <https://www.youtube.com/watch?v=TVfl2KEkq5A> and <https://journeynorth.org/t/m/crane/jr/JnKidsOverview.html>.

Bacterial Population Growth

A single bacterium is put in a container with plenty of food for bacteria. After 30 minutes the bacterium divides into two bacteria. Then, every 30 minutes, each bacterium in the container divides in two. As a result, population size doubles every 30 minutes.



3. Add to this figure to show how population size will double from 60 minutes to 90 minutes.

4. Complete this table to show how many bacteria there will be at each time if the number of bacteria doubles every 30 minutes. Then, fill in the “Increase in # bacteria” blanks.

Time (minutes)	0	30	60	90	120	150	180	210	240	270	300
# Bacteria	1	2									

Increase in # bacteria _____

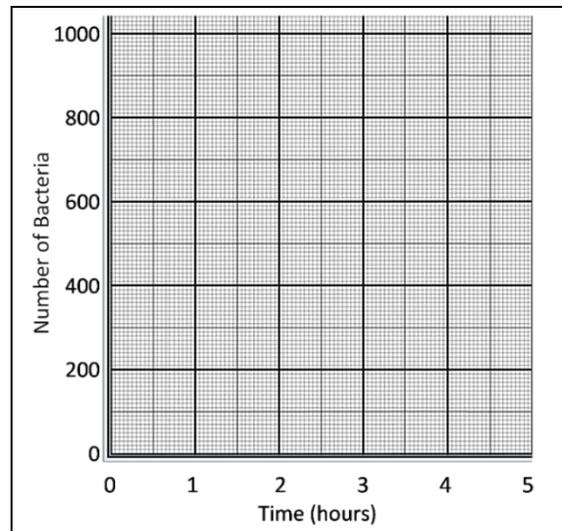
Notice that, for each 30-minute interval, $\Delta N = N$, where

- ΔN (pronounced delta N) represents the change in the number of bacteria from the beginning to the end of a 30-minute interval
- N represents the number of bacteria at the beginning of the 30-minute interval

5. Explain the biological reason why $\Delta N = N$ for each 30-minute interval.

6a. Graph the number of bacteria at each time. Connect the points to show the population growth curve.

6b. Why did population size increase slowly at the beginning and more rapidly at later times?



Exponential population growth occurs when the rate of population growth increases as population size increases (ΔN is proportional to N). This bacterial population showed exponential population growth.

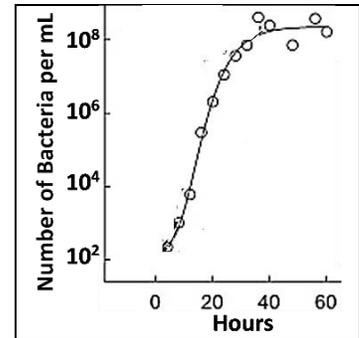
7. What evidence supports the conclusion that the whooping crane population also showed roughly exponential population growth? (Hint: See the table on the previous page.)

8a. Can exponential growth of the bacterial population continue forever? yes ___ no ___

8b. Why or why not?

Limits on Population Growth

9a. This graph shows that, during the first twenty hours, population size increased from a few hundred bacteria to about ten million bacteria. After that, population growth slowed and then stopped. What are some possible reasons why population growth stopped for these bacterial populations?



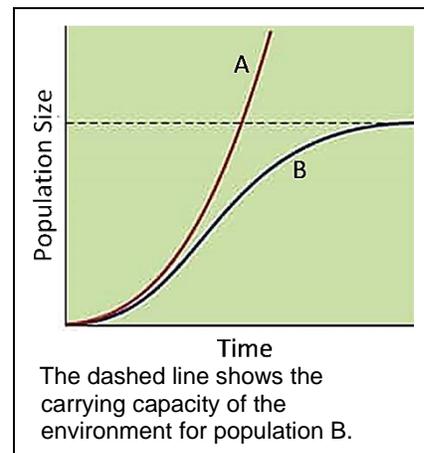
9b. As population size increases, competition for limited resources _____ . An increase in competition results in increased mortality and/or decreased reproduction, so population growth slows and stops. As a result, population size reaches a _____ , which is called the **carrying capacity** of the environment. This type of population growth is called **logistic population growth**. In contrast, exponential population growth occurs when resources are so abundant that there is little or no competition for resources.

10a. This figure shows the growth of two populations of bacteria which lived in different environments. The amount of resources in the environment was

- much bigger for population A.
- much bigger for population B.
- about the same for both populations.

10b. Population A shows _____ population growth, and population B shows _____ population growth.

10c. Notice that the difference between the exponential and logistic population growth curves is small at the beginning and gets bigger at later times. Explain why.



11. Different limiting factors can determine the carrying capacity of an environment. For example, the carrying capacity for bluebirds could be determined by the amount of food or the number of holes where a bluebird pair can build a nest. Describe an experiment that could test whether food or the number of nest holes limits the size of bluebird populations.



This table compares the mathematical equations for exponential vs. logistic population growth for bacteria.

Exponential Population Growth	Logistic Population Growth
$\Delta N = N$ where <ul style="list-style-type: none"> • ΔN is the change in the number of individuals from the beginning to the end of a time interval • N is the number of individuals at the beginning of the time interval 	$\Delta N = N \frac{(K - N)}{K}$ where <ul style="list-style-type: none"> • ΔN and N are the same as for exponential growth • K is the carrying capacity of the environment = the number of individuals that can live in the environment

12. Notice that the first part of the equation for logistic population growth is the same as the equation for exponential population growth. Circle the algebraic expression that has been added in the logistic population growth equation.

13. Fill in each blank in the paragraph below with the best match from these algebraic expressions: K $K - N$ $(K - N)/K$

When there are N individuals in the population, the number of additional individuals that can live in the environment is _____. Therefore, the fraction of carrying capacity that is still available for population growth is _____.

14a. Calculate $(K - N)/K$ for $K = 100$ and $N = 1$.

14b. Calculate $(K - N)/K$ for $K = 100$ and $N = 99$.

14c. Complete the following table.

	Provide an explanation based on the equations for exponential and logistic population growth.	Provide a biological explanation.
Explain why logistic population growth is similar to exponential population growth when N is small.		
Explain why logistic population growth slows to nearly 0 as N gets close to K .		

15. The equations (above) and the figure (in question 10) are two different versions of the exponential and logistic population growth models. Give one advantage of each version.

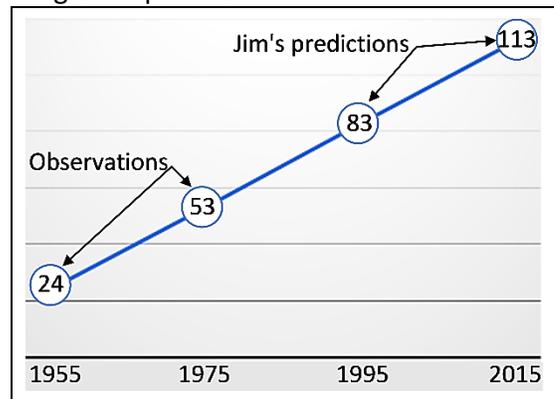
Advantage of the figure:

Advantage of the equations:

Using the Exponential and Logistic Population Growth Models to Understand Recovery of Endangered Species

A **model** is a simplified representation of reality that can help us to understand a real-world phenomenon. For example, the exponential and logistic population growth models help us to understand and predict changes in population size for endangered species.

In 1975, Jim predicted the future growth in the number of whooping cranes in Texas. He knew that this number had increased from about 24 in 1955 to about 53 in 1975. Since population size had increased by about 30 in the past twenty years, Jim predicted that population size would increase by about 30 every twenty years in the future. Thus, Jim predicted that the number of whooping cranes would reach about 113 by 2015. We now know that the actual population size in 2010-2015 was about 290, more than double Jim's prediction.



16. What was wrong with Jim's reasoning? Why was his prediction so much lower than actual population growth?

17a. Conservation biologists have used two approaches to prevent the extinction of whooping cranes. They helped the one surviving population of wild whooping cranes to increase in size, and they also established additional populations of whooping cranes in other locations. Conservation biologists expected that, eventually, the total number of wild whooping cranes would be greater if there were several populations in different wetlands, instead of a single population in one wetland. Explain why. (Hint: Think about carrying capacity.)

17b. What is another reason why there is less risk of extinction if there are several populations of whooping cranes at different locations and not just one population in a single location?

The Endangered Species Act (ESA) was passed in 1973 to prevent the extinction of endangered species in the US. Critics argue that the ESA has been ineffective since, so far, less than 2% of all protected species have recovered enough that they no longer need ESA protection. In contrast, supporters of the ESA argue that population growth is expected to take a long time, especially when recovery efforts start from a very small population and the endangered species has a low birth rate (e.g., about 1 chick per pair of whooping cranes per year). For many endangered species, population sizes have increased at expected rates, with reasonable progress toward expected future recovery.

18. In 2021, after 70 years of conservation efforts, the number of wild whooping cranes had increased from about 20 in a single population to about 700 in several different populations. However, whooping cranes are still thought to be at risk for extinction and are still protected by the ESA. Would you describe the conservation efforts for whooping cranes as a success or a failure? Explain your reasons.

Exponential and Logistic Population Growth Models vs. Complex Reality

The exponential and logistic population growth models can explain some trends in population size, but not others. For example, page 1 describes two major trends in numbers of whooping cranes.

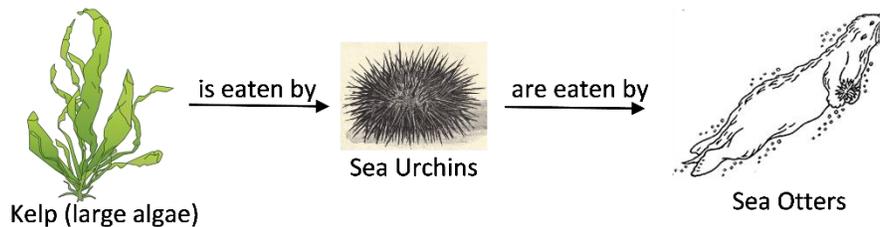
- A. Before about 1950, the number of whooping cranes decreased drastically and many populations of whooping cranes were entirely eliminated by hunting and habitat loss.
- B. After about 1950, the surviving population of whooping cranes increased in size, slowly at first and then more rapidly.

19a. Which of these trends could be predicted or explained by the exponential population growth model? A ___ B ___

19b. Does either the exponential or logistic population growth model ever predict a decrease in population size? yes ___ no ___

Notice that neither the exponential nor the logistic population growth model includes the effects of changes in the environment such as increased predation or habitat loss. Therefore, neither model can predict or explain the decrease in whooping crane populations before 1950.

Human hunting has decreased population size for many species. This can have indirect effects on population size for other organisms in the same ecosystem. For example, hunting drastically reduced the populations of sea otters, so sea otters were rare at the beginning of the twentieth century. This affected population size for the other organisms in this food chain.



20a. Predict the effect of a decrease in sea otter population size on sea urchin population size. Explain your reasoning.

20b. Predict the effect of a decrease in sea otter population size on the abundance of kelp. Explain your reasoning.

21. To summarize what you have learned thus far, fill in one match per blank. (You may use each match more than once.)

The exponential population growth model includes ____

The logistic population growth model includes ____ ____

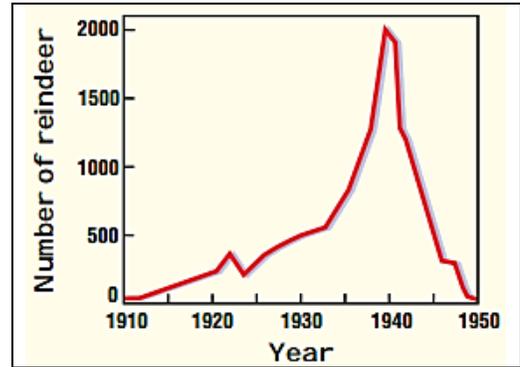
In the real world, population size can be affected by ____ ____ ____

- a. increases in population size as a result of reproduction
- b. the effects of changes in the environment
- c. the effects of competition for limited resources

Models can help us understand and predict biological phenomena such as changes in population size. However, all models are based on simplifying assumptions and when these assumptions are not accurate the predictions of the model are not accurate. For example, the predictions of the logistic population growth model are only accurate if:

- carrying capacity is constant (e.g., no habitat destruction);
- as population size approaches carrying capacity, population growth slows promptly, so the population does not exceed the carrying capacity.

This graph illustrates what can happen when these simplifying assumptions are not true. In 1911, 25 reindeer were brought to an island off the coast of Alaska. Initially, food was plentiful, and the number of reindeer increased. By the late 1930s the large population of reindeer had drastically reduced the amount of lichen (which the reindeer depended on for their winter food). The reindeer population plummeted in the 1940s, but the slow-growing lichen had not recovered by 1950.



22a. Scientists believe that originally the carrying capacity of the island was about 700 reindeer. Draw the expected trend in population size if population growth followed the logistic population growth model with a carrying capacity of 700 reindeer; start from the actual population size in the early 1930s.

22b. By 1950, the number of reindeer had fallen well below the original carrying capacity of 700. Explain why population size fell so low.

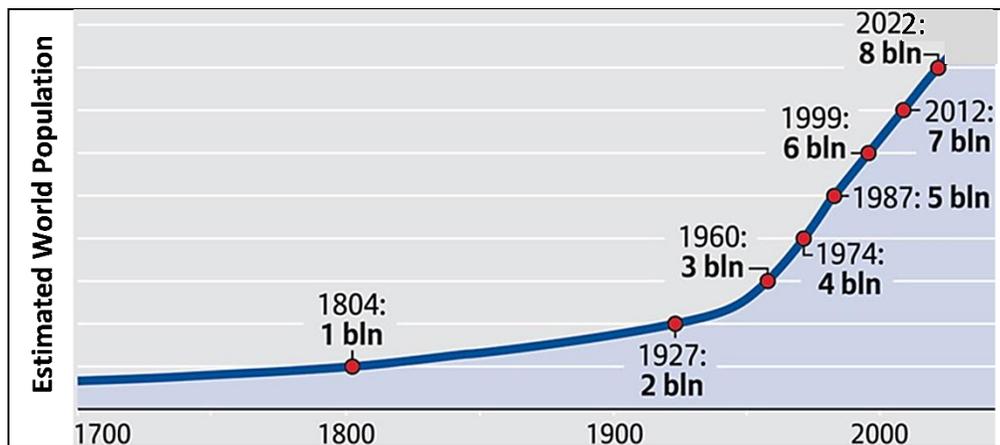
22c. Why didn't the logistic population growth model accurately predict trends in the size of this reindeer population?

23a. Dr. Smart declared, "The exponential and logistic population growth models are useless. I have developed a new model that can accurately predict changes in population size for any population anywhere." Biologists disagree with both sentences. Give examples that illustrate how the exponential and logistic population growth models are useful.

23b. Explain why we should not believe Dr. Smart's boast that his model will be accurate for all populations.

Human Population Growth

We have defined a population as a group of individuals of the same species that live in the same area at the same time. In contrast, in this section, we will discuss changes in the total number of people on the Earth.



24. It took many thousands of years for world population to reach 1 billion people. Then, it took only a little over a century for world population to add 1 billion more people. After that, world population increased rapidly – by 6 billion in less than a century. Explain how the exponential population growth model helps us to understand the trends in world population growth.

25. Can you conclude from the above graph that the Earth's carrying capacity is at least 8 billion people? Explain your reasoning. (Hint: Remember the reindeer.)

What is the Earth's carrying capacity for humans? In other words, how many people can the Earth support in the long-term? Scientists disagree about the answer to this question, but all agree that the Earth's carrying capacity will be influenced by people's behavior.

26. For example, the Earth's carrying capacity is influenced by the amount and type of food that people consume. The average person in the US consumes approximately 25% more calories and almost three times as much meat as the average person in the world. Suppose that the average consumption per person in other parts of the world increased to equal the US average. How would this change the Earth's carrying capacity for humans? Explain your reasoning.